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Sigma terms from an SU(3) chiral extrapolation

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We report a new analysis of lattice simulation results for octet baryon masses in 2+1-flavor QCD, with an emphasis on a precise determination of the strangeness nucleon sigma term. A controlled chiral extrapolation of a recent PACS-CS Collaboration data set yields baryon masses which exhibit remarkable agreement both with experimental values at the physical point and with the results of independent lattice QCD simulations at unphysical meson masses. Using the Feynman-Hellmann relation, we evaluate sigma commutators for all octet baryons. The small statistical uncertainty, and considerably smaller model-dependence, allows a significantly more precise determination of the pion-nucleon sigma commutator and the strangeness sigma term than hitherto possible, namely $\sigma_{\pi N} = 45 \pm 6$ MeV and $\sigma_s = 21 \pm 6$ MeV at the physical point.

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The light-quark sigma terms provide critical information concerning the nature of explicit chiral symmetry breaking in QCD, as well as the decomposition of the mass of the nucleon [1]. While these physical observables are difficult to measure with conventional probes, an accurate knowledge of the sigma terms is of essential importance in the interpretation of experimental searches for dark matter [2–6]. Dark matter candidates, such as the favoured neutralino, a weakly interacting fermion with mass of order 100 GeV or more, have interactions with hadronic matter which are essentially determined by couplings to the light and strange quark sigma commutators.

Experimentally, $\sigma_{\pi N}$ is determined from πN scattering through a dispersion relation analysis. Traditionally, the strange scalar form factor has then been evaluated indirectly using $\sigma_{\pi N}$ and a best-estimate for the non-singlet contribution $\sigma_0 = m_l\langle N|\overline{u}u + \overline{d}d - 2\overline{s}s|N\rangle$. These traditional evaluations have yielded a value for σ_s as large as 300 MeV, compared to 50 MeV for the light quark commutator, indicating that as much as one third of the nucleon mass might be attributed to non-valence quarks. This suggestion appears to be incompatible with widely-used constituent quark models, and has generated much theoretical interest over the last two decades.

The traditional method of determination of σ_s is severely limited because it involves the small difference between $\sigma_{\pi N}$ (with its uncertainty) and σ_0 which is usually deduced in terms of $SU(3)$ symmetry breaking. Even given a perfect determination of $\sigma_{\pi N}$, σ_s will have an uncertainty of order ~ 90 MeV [7]. For that reason σ_s has been considered notoriously difficult to pin down. In recent years, the best value for σ_s has seen an enormous revision. Advances in lattice QCD have revealed a strange sigma term of 20-50 MeV [8–17], an order of magnitude smaller than was previously believed.

In this Letter we use the finite-range regularisation (FRR) technique to effectively resum the chiral perturbation theory expansion of the quark mass dependence of octet baryons. Fitting the resulting functions to recent lattice data, we extract the scalar form factors by simple differentiation using the Feynman-Hellmann theorem. Our technique allows comparison with recent direct lattice QCD calculations of the flavor-singlet matrix elements at unphysical meson masses [19], with consistent results. We report values of $\sigma_{\pi N} = 45 \pm 6$ MeV and $\sigma_s = 21 \pm 6$ MeV at the physical point.

The sigma terms of a baryon B are defined as scalar form factors, evaluated in the limit of vanishing momentum transfer. For each quark flavor q,

$$\sigma_{Bq} = m_q\langle B|\overline{q}q|B\rangle; \quad \overline{\sigma}_{Bq} = \sigma_{Bq}/M_B. \tag{1}$$

For the nucleon, the so-called πN sigma commutator and the strange sigma commutator are defined by

$$\sigma_{\pi N} = m_l\langle N|\overline{u}u + \overline{d}d|N\rangle, \tag{2}$$

$$\sigma_s = m_s\langle N|\overline{s}s|N\rangle, \tag{3}$$

where $m_l = (m_u + m_d)/2$.

Following the technique described in Refs. [8, 20], we fit octet baryon mass data recently published by the PACS-CS Collaboration [21] using a chiral expansion:

$$M_B = M^{(0)} + \delta M_B^{(1)} + \delta M_B^{(3/2)} + \ldots \tag{4}$$

Here, $M^{(0)}$ denotes the degenerate mass of the baryon octet in the SU(3) chiral limit, $\delta M_B^{(1)}$ gives the correction linear in the quark masses, and $\delta M_B^{(3/2)}$ represents quantum corrections corresponding to one-loop contributions from the pseudo-Goldstone bosons $\phi = \pi, K, \eta$. Explicit expressions for the extrapolation formulae and for renormalisation of the loop integrals may be found in Ref. [20].

Following Ref. [8], we retain the octet-decuplet mass difference δ in numerical evaluations to properly account for the branch structure near $m_\phi \sim \delta$. The loop contribution parameters are set to appropriate experimental and

phenomenological values; $D+F=g_A=1.27$, $F=\frac{2}{3}D$, $C=-2D$, $f=0.0871$ GeV, and $\delta=0.292$ GeV. Within the framework of FRR, we introduce a mass scale Λ, through a regulator $u(k)$. Λ is related to the scale beyond which a formal expansion in powers of the Goldstone boson masses breaks down. In practice, Λ is chosen by fitting to the lattice data itself. For further discussions of the FRR regularization scheme, we refer to Refs. [22–26]. To provide an estimate of the model-dependent uncertainty in our result, we consider a variety of forms of the regulator $u(k)$, namely monopole, dipole, and Gaussian, as well as a sharp cutoff. To further estimate systematic uncertainties, we allow f, the meson decay constant in the chiral limit, the baryon-baryon-meson coupling constants F and C, and δ to vary by $\pm 10\%$ from the central values given above; see Ref. [27] for details. The effect of these variations are included in the final quoted errors.

The PACS-CS results have been corrected for small, model-independent, finite volume effects before fitting. These finite volume corrections were evaluated by considering the leading one-loop results of chiral EFT [8, 28–30]. We note that the largest shift was -0.022 ± 0.002 GeV for the nucleon at the lightest pion mass.

The fit to the PACS-CS baryon octet data is shown in Figure 1. We find an optimal dipole regularization scale of $\Lambda=0.9\pm 0.1$ GeV, in close agreement with the value deduced from an analysis of nucleon magnetic moment data [31] and, from the phenomenological point of view, remarkably close to the value preferred from comparison of the nucleon's axial and induced pseudoscalar form factors [32]. The minimum $\chi^2_{\rm dof}$ is 0.41 (6.1/(20-5)) for the dipole, and varies between 0.40 and 0.42 for the other regulators. This value is somewhat lower than unity, as correlations between the lattice data cannot be accounted for without access to the original data.

Clearly, the fit is very satisfactory over the entire range of quark masses explored in the simulations. Furthermore, the masses of the octet baryons agree remarkably with experiment at the physical point. A comparison of the extrapolated baryon masses with the best experimental values is given in Table I. The first error quoted is statistical and includes the correlated uncertainty of all of the fit parameters including the regulator mass Λ, while the second is an estimate of model-dependence. This includes the full variation over dipole, monopole, sharp cutoff and Gaussian regulator forms, as well as accounting for the variation of the phenomenologically-set parameters F, C and δ described earlier.

As we fit baryon mass functions to lattice data over a range of pseudoscalar masses significantly larger than the physical values, it is prudent to check the consistency of our results as the analysis moves outside the power-counting regime (PCR), where higher order terms may become significant. By performing our fit to progressively fewer data points, that is, by dropping the heaviest mass points, we test the scheme dependence of our evaluation.

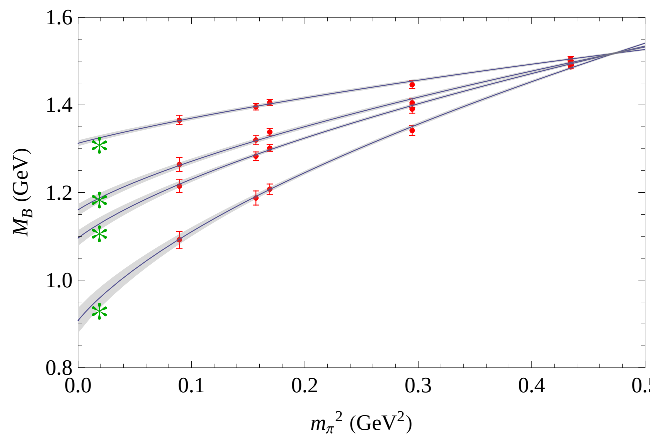

FIG. 1. Fit to the PACS-CS baryon octet data. Error bands shown are purely statistical, and incorporate correlated uncertainties between all fit parameters. Note that the data shown has been corrected for finite volume and the simulation strange quark mass, which was somewhat larger than the physical value. The green stars show experimental values.

The results are consistent, and largely independent of the truncation of the data. This can be seen clearly in Figure 2, which shows the variation of the dimensionless baryon sigma terms as progressively fewer data points are used for the fit to the octet masses. The points shown correspond to an evaluation with a dipole regulator, and error bars are purely statistical.

B	Mass (GeV)	Experimental	$\overline{\sigma}_{Bl}$	$\overline{\sigma}_{Bs}$
N	0.959(24)(9)	0.939	0.047(6)(5)	0.022(6)(0)
Λ	1.129(15)(6)	1.116	0.026(3)(2)	0.141(8)(1)
Σ	1.188(11)(6)	1.193	0.020(2)(2)	0.172(8)(1)
Ξ	1.325(6)(2)	1.318	0.0089(7)(4)	0.239(8)(1)

TABLE I. Extracted masses and sigma terms for the physical baryons. The first uncertainty quoted is statistical, while the second results from the variation of various chiral parameters and the form of the UV regulator as described in the text. The experimental masses are shown for comparison.

To further test our claim that the fitted mass functions accurately describe the variation of the baryon masses with quark mass, we compare our extrapolation with independent lattice data along a very different trajectory in the m_l-m_s plane, as compared to the fit domain. Most lattice simulations, including that of the PACS-CS Collaboration, hold the simulation strange quark mass fixed near the physical value, and progressively lower the light quark mass to approach the physical point. However, the UKQCD-QCDSF Collaboration has recently presented an alternative method of tuning the quark masses, in which the singlet mass $(2m_K^2+m_\pi^2)$ is held fixed [33]. This procedure constrains the simulation kaon mass to

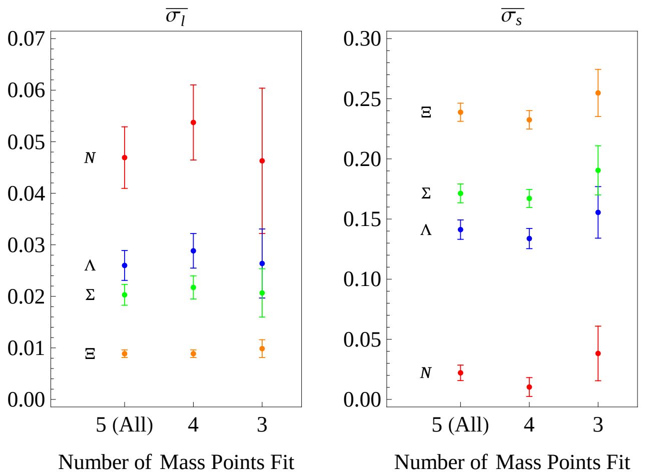

FIG. 2. Dimensionless baryon sigma terms, evaluated using a dipole regulator, based on fits to the NPLQCD results at the lightest 5 (all), 4, and 3 pseudoscalar mass points.

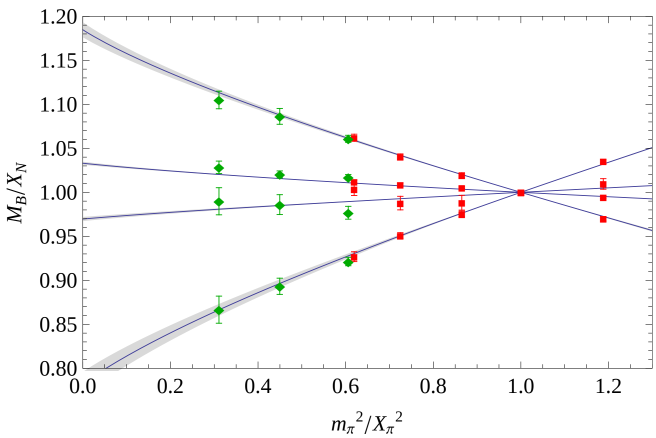

FIG. 3. Prediction of UKQCD-QCDSF lattice data, based on our fit to the PACS-CS octet baryon mass simulation. Red (square) and green (diamond) points correspond to 24^3 and 32^3 lattice volumes respectively. Error bands shown are purely statistical, and incorporate correlated uncertainties between all fit parameters.

always be less than the physical value. In comparison, the traditional trajectory in the $m_\pi - m_K$ plane necessarily keeps the kaon mass larger than the physical value.

The close match between our fit to the PACS-CS points and the UKQCD-QCDSF lattice data, shown in Figure 3, is extremely encouraging. We emphasize that the lines in Figure 3 are *not* a fit to the data shown, but rather a prediction, resulting from the described fit to the PACS-CS octet data being evaluated along the UKQCD-QCDSF simulation trajectory.

All lattice points shown in Figure 3 have been shifted, by the procedure described for the PACS-CS data, to account for finite-volume effects. We chose to use the lattice

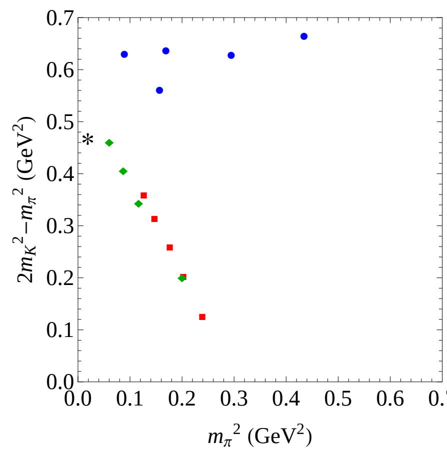

FIG. 4. Locations of lattice QCD simulations by the PACS-CS Collaboration (blue circles), and UKQCD-QCDSF Collaboration (red and green squares and diamonds) in the $m_l - m_s$ plane. The star denotes the physical point. Figure 3 shows the fit to the PACS-CS data only, evaluated at the UKQCD-QCDSF simulation quark masses.

spacing $a = 0.078$ fm deduced by the UKQCD-QCDSF Collaboration. For further details of the UKQCD-QCDSF data set, and the normalizations X_N, X_π, we refer to Ref. [33].

Figure 4 illustrates the significance of the prediction shown in Figure 3. While our fit was made only to the PACS-CS data, it successfully reproduces the UKQCD-QCDSF lattice results, at points in the $m_l - m_s$ plane which are substantially different from the PACS-CS simulation trajectory. This very strongly supports our claim that the sigma terms, which correspond to derivatives in the plane shown in Figure 4, are accurately determined by our fit.

To extract the sigma commutators from our baryon mass functions, we use the Feynman-Hellman relation [34],

$$\sigma_{Bq} = m_q \frac{\partial M_B}{\partial m_q}, \tag{5}$$

and, as above, replace quark masses by meson masses squared: $m_l \to m_\pi^2/2$ and $m_s \to (m_K^2 - m_\pi^2/2)$. For the case of the nucleon, we recall the alternative conventional notation to quantify the strangeness content, namely the kaon sigma term

$$\sigma_{KN} = \frac{1}{2}(m_l + m_s)\langle N|\bar{l}l + \bar{s}s|N\rangle. \tag{6}$$

As in Equation 1, overlines indicate corresponding (dimensionless) quantities normalized by the baryon mass.

A direct measure of the magnitude of the strange quark content of the nucleon relative to its light quark content:

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{m_l}{m_s}\frac{2\sigma_s}{\sigma_{\pi N}}, \tag{7}$$

can be trivially evaluated given the strange and light quark sigma terms. At the physical point, we find $\sigma_{\pi N} = 45 \pm 6$ MeV, $\sigma_{KN} = 300 \pm 40$ MeV and $\sigma_s = 21 \pm 6$ MeV, corresponding to a y-value of 0.04 ± 0.01 for $m_l/m_s = 0.039(6)$ [35]. This analysis also constrains $\sigma_{\pi N} - \sigma_0$ to be 1.64 ± 0.53 MeV. The quoted errors include all systematic and model-dependent uncertainties combined in quadrature. Results for the other octet baryons are made explicit in Table I.

An advantage of the method used here is that we can easily evaluate sigma terms from our fit at any pion or kaon mass. The QCDSF Collaboration has recently presented very precise direct calculations of $\sigma_{\pi N}$ and σ_s from lattice QCD, at light quark masses somewhat larger than the physical values [19]. At (m_π, m_K) values of $(281, 547)$ MeV, the Collaboration quotes $\sigma_{\pi N}^{\mathrm{QCDSF}} = 106(11)(3)$ MeV, and $\sigma_s^{\mathrm{QCDSF}} = 12^{+23}_{-16}$ MeV. This compares very favorably to the results of our fits at these particular pseudoscalar masses, namely $\sigma_{\pi N} = 131(11)(5)$ MeV and $\sigma_s = 16(5)(1)$ MeV. Once again, the two uncertainties correspond to the described evaluation of systematic and model-dependent uncertainties.

The conclusion of our analysis is clear. By developing closed-form functions for baryon mass as a function of quark mass based on a fit to PACS-CS Collaboration lattice data, we were able to determine precise baryonic sigma terms by simple differentiation. This method allows us to achieve small statistical and model-dependent uncertainties. Moreover, we find excellent agreement with recent direct lattice calculations of these values at unphysical pseudoscalar masses. Decidedly our most significant result is a very precise value for the strangeness nucleon sigma term, namely $\sigma_s = 21 \pm 6$ MeV at the physical point.

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